ASYMPTOTIC CALCULATION OF THE CATHODE LAYER IN A MOLECULAR-GAS PLASMA

M. S. Benilov and Z. G. Kamalov

UDC 533.9.082.76

A continuum calculation of the current-voltage characteristic of the boundary layer of a weakly ionized molecular-gas plasma at a nonemitting cathode, taking into account the dependence of the transport and kinetic coefficients of the plasma on the electric field intensity, was performed in [1]. In this paper the impedance of such a boundary layer is calculated.

The problem of calculating the impedance of the collisional cathode region taking into account nonstationary effects in the Debye layer was previously studied in [2-5] for a chemically frozen Debye layer and in [6] for a Debye layer with ionization by an external source. An approximate analytic solution was constructed with the help of a priori separation of the perturbation region into a Debye layer and a quasineutral region with some joining conditions (also a priori) at the (nonstationary) outer boundary of the Debye layer. In this paper the method of joined asymptotic expansions is employed to solve both the nonlinear stationary problem and the linear problem for perturbations.

1. Formulation of the Problem. We shall study a gas-dynamic boundary layer of weakly ionized plasma on a flat electrically conducting surface. The plasma contains neutral particles and singly charged positive ions and electrons. We shall use the hydrodynamic system of equations for the distributions of the molar fractions x_i and x_e and diffusion flux densities J_i and J_e for ions and electrons and the electric field intensity. By analogy to [7] we shall write this system in the form

$$J_i = -nD_i\left(\frac{\partial x_i}{\partial y} - x_i\frac{eE^0}{kT}\right), \quad J_e = -n D_e\frac{\partial x_e}{\partial y} - nx_e\mu_e^0E^0,$$

$$n\frac{\partial x_i}{\partial t^0} + \frac{\partial J_i}{\partial y} = f_{i1} - k_{r1}n_in_e + k_{i2}n_e,$$

$$j^0 = e(J_i - J_e) + \frac{1}{4\pi}\frac{\partial E^0}{\partial t^0}, \quad \frac{\partial j^0}{\partial y} = 0, \quad \frac{\partial E^0}{\partial y} = 4\pi en(x_i - x_e).$$

Here t_0 is the time; the y axis is oriented along the normal from the wall; n is the particle density in the plasma; $n_i = nx_i$, $n_e = nx_e$ are the ion and electron densities ($n_i \ll n$, $n_e \ll n$); D_i and D_e are the coefficients of diffusion of ions and electrons; μ_e^0 is the mobility of electrons; j^0 is the total electric current density (including the condution and displacement currents); T is the heavy-particle temperature; e is the electron charge; k is Boltzmann's constant; the terms on the right side of the third equation take into account, following [1], the stepped ionization with the participation of heavy particles, the corresponding inverse process (recombination), and direct ionization by electron impact (this process is significant for sufficiently large values of E^0); f_{i1} is the rate of stepped ionization; k_{r1} is the coefficient of stepped recombination; and, $k_{i2}n_e$ is the rate of direct ionization. It is assumed that the mobility and coefficient of diffusion of the ions are related by Einstein's relation, and the convective transport of charged particles is assumed to be negligibly small compared with the volume ionization.

For simplicity we shall neglect the dependence of the coefficients of diffusion of ions and electrons on E^0. We shall also neglect the dependence of the coefficient of stepped recombination on E^0; then the representation $f_{i1} = k_{r1}n_{er}^2$, where n_{er} is the chemically equilibrium quasineutral charged-particle density in the limit of a weak field, is valid. Thus D_i, D_e, k_{r1}, n_{er} are assumed to be given functions of the local values of T and n and the partial composition of the neutral components; μ_e^0 and k_{i2} are given functions of the same argu

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 41-49, January-February, 1988. Original article submitted October 8, 1986.

 0021-8944/88/2901-0038$12.50

ments and E^0. In the concrete calculations performed below the dependences from [1] are employed for these functions.

We shall assume that the Joule heating in the boundary layer is much weaker than the convective heat flux, and we shall neglect the effect of ionization on the flow field of the neutral component. Then the functions T and n and the partial composition of the neutral components can be found by solving the corresponding gas-dynamic problem neglecting the presence of ionization; in this formulation, the functions of the particle density on the surface of the electrode, which we assume to be ideally absorbing, catalytic, and nonemitting, may be assumed to equal zero; at distances much greater than the thickness of the boundary layer δ, the concentrations approach the fixed value $n_{er\infty}$ (the subscript ∞ is assigned to values of the corresponding functions on the outer boundary of the boundary layer):
$x_i = x_e = 0,\ y/\delta \to \infty:\ x_i \to n_{er\infty}/n_\infty,\ x_e \to n_{er\infty}/n_\infty.$

For the missing boundary condition we fix the value of the electric current density j^0. We shall transform the formulated problem to dimensionless variables:

$$I_i = -a\left(z_i' - \Theta^{-1} z_i E\right), \quad I_e = -a z_e' - \mu z_e E; \tag{1.1}$$

$$\frac{\alpha}{\Theta}\frac{\partial z_i}{\partial t} + I_i' = 2b\chi^{-1}\left(r^2 - z_i z_e + c z_e\right); \tag{1.2}$$

$$I_e = \beta\left(I_i - \frac{j}{\chi} + \varepsilon\alpha\frac{\partial E}{\partial t}\right) \quad (j = j(t) \text{ problem}); \tag{1.3}$$

$$\varepsilon\Theta E' = z_i - z_e; \tag{1.4}$$

$$\eta = 0:\ z_i = z_e = 0,\ \eta \to \infty:\ z_i \to 1,\ z_e \to 1, \tag{1.5}$$

$$\eta = \frac{y}{\delta},\quad t = \lambda t^0,\quad E = \frac{e\delta E^0}{kT_\infty},\quad z_m = \frac{x_m n_\infty}{n_{er\infty}},\quad I_m = \frac{\delta J_m}{D_{m\infty} n_{er\infty}}$$

$$(m = i, e),\quad a = \frac{nD_i}{n_\infty D_{i\infty}} = \frac{nD_e}{n_\infty D_{e\infty}},\quad \Theta = \frac{T}{T_\infty},\quad \mu = \frac{kT_\infty n \mu_e^0}{eD_{e\infty} n_\infty},$$

$$b = \frac{k_{r1} n^2}{k_{r1\infty} n_\infty^2},\quad r = \frac{n_{er} n_\infty}{n_{er\infty} n},\quad c = \frac{k_{i2} n_\infty}{k_{r1} n_{er\infty} n},\quad \alpha = \delta^2\lambda/D_{i\infty},$$

$$\chi = \frac{2D_{i\infty}}{k_{r1\infty} n_{er\infty}\delta^2},\quad \beta = \frac{D_i}{D_e},\quad \varepsilon = \frac{kT_\infty}{4\pi n_{er\infty} e^2\delta^2},\quad j = \frac{\chi\delta j^0}{eD_{i\infty} n_{er\infty}}.$$

Here it is assumed for simplicity that the ratio of the coefficients of diffusion of ions and electrons is constant in the volume of the boundary layer; the function n/n_∞ on the left sides of (1.2) and (1.4) is written, in accordance with the condition that the pressure is constant across the boundary layer, as Θ^{-1}, λ is the inverse characteristic time of the perturbations of interest; the prime denotes differentiation with respect to η. Based on the foregoing, Θ, a, b, r, c, μ are assumed to be given functions:

$$\Theta = \Theta(\eta),\ a = a(\eta),\ b = b(\eta),\ r = r(\eta),\ c = c(\eta, E),\ \mu = \mu(\eta, E). \tag{1.6}$$

We shall assume that the "given" function j(t) contains a stationary component and a small component depending exponentially on the time (perturbation) $j(t) = j^s + \nu j^p \exp(-\lambda t^0) = j^s + \nu j^p \exp(-t)$, where j^s, j^p, and ν are given parameters; $j^p = O(j^s)$; $\nu = o(1)$. The index in the exponential is in general complex (λ is a complex quantity; in addition, together with purely imaginary values, in connection with the analysis of stability problems we shall also study its values with a nonzero real part).

We shall seek the solution of the problem in an analogous form. For functions with an index s, describing the stationary base state, we have a problem analogous to (1.1)-(1.6) (without nonstationary terms; instead of z_m, I_m, E, j, μ, c we shall have z_m^s, I_m^s, E^s, j^s, μ^s, c^s). For functions with the index p, describing perturbations caused by the perturbation of the current density, we obtain the linearized problem.

2. Asymptotic Formulation. The stationary problem and the problem for perturbations contains dimensionless determining parameters ε, χ, β, j^s, j^p, α. Under typical conditions [1] the first three of them are of the order of 10^{-8}, 10^{-4}, 10^{-2}, respectively, and are assumed to be small in the asymptotic analysis. We note that the smallness of ϵ and χ indicates quasineutrality and ionization equilibrium in the outer part of the boundary layer, and the smallness of β takes into account the difference in the drift velocities of the ions and

electrons. We shall establish the following order ratios between ϵ, χ, and β: $\varepsilon/\chi \to 0$, $\beta = k_1\chi^{1/2}$, where k_1 is fixed. We assume that the parameter j^s is negative and fixed, which corresponds to the case of a thick Debye layer at the cathode with intense generation of charged particles [7]. The parameter j^p, in accordance with the foregoing, is assumed to be comparable to j^s, and therefore also fixed. The parameter α can vary over a wide range: in this work we examine the cases $\alpha = O[(\varepsilon\chi)^{-1/2}]$, $\alpha = O(\varepsilon^{-1/2}\chi^{-1})$, corresponding to perturbations with characteristic times of the order of the drift times for ions or electrons drifting through the Debye layer.

According to the results of [1], direct ionization and the dependence of μ_e^0 on E^0 become significant only in the Debye layer; outside this layer the rate of direct ionization is exponentially low, $\mu_e^0 = eD_e(kT)^{-1}$. For this reason, we assume that the functions c and μ have the form $c = \varepsilon^{-1/2}w_1(\varepsilon\chi E^2)$, $\mu = a\Theta^{-1}w_2(\varepsilon\chi E^2)$, where $w_1 = w_1(x)$, $w_2 = w_2(x)$ are given functions, the first of which decays to zero exponentially and the second approaches unity (they also depend on η).

3. Asymptotic Solution.

We shall study perturbations with a characteristic time of the order of the drift time of ions drifting through the Debye layer: $\alpha = (\varepsilon\chi)^{-1/2}\alpha_1$, $\alpha_1 = O(1)$. We shall write the asymptotic expansions of the solutions of the stationary problem and the perturbation problem, valid in the outer (equilibrium) part of the Debye layer, as

$$z_i^s = r + \ldots,\quad z_e^s = r + \ldots,\quad I_i^s = \chi^{-1/2}k_1 j^s + \ldots,$$
$$I_e^s = -\chi^{-1/2}k_1 j^s + \ldots,\quad E^s = \chi^{-1/2}k_1\Theta j^s/(ar) + \ldots,$$
$$z_i^p = (-\varepsilon^{3/2}\chi^{-1}2br\Theta/\alpha_1 + \varepsilon k_1)\,k_1\Theta j^p\,(\Theta a^{-1}r^{-1})' + \ldots,$$
$$z_e^p = -\varepsilon\chi^{-1/2}k_1\Theta j^p\,(\Theta a^{-1}r^{-1})' + \ldots,\quad I_i^p = \chi^{-1/2}k_1 j^p + \ldots,$$
$$I_e^p = -\chi^{-1/2}k_1 j^p + \ldots,\quad E^p = \frac{\chi^{-1/2}\Theta k_1 j^p}{ar} + \ldots,\quad \eta_D < \eta < \infty.$$

The thickness of the volume charge layer introduced here η_D is a function of the current density, and must be determined in the course of the solution of the stationary problem.

The expansions valid in the first transitional layer are

$$z_i^s = g_2(\eta_2) + \ldots,\quad z_e^s = g_2(\eta_2) + \ldots,$$
$$I_i^s = \chi^{-1/2}(k_1 j^s - 2a_D dg_2/d\eta_2) + \ldots,\quad I_e^s = -\chi^{-1/2}k_1 j^s + \ldots,$$
$$E^s = \chi^{-1/2}\Theta_D\,(k_1 j^s - a_D dg_2/d\eta_2)/(a_D g_2) + \ldots,$$
$$z_i^p = \left(\frac{\varepsilon}{\chi}\right)^{3/2}\frac{k_1\Theta_D^3 j^p}{\alpha_1}\left[\frac{d^2}{d\eta_2^2}\left(\frac{k_1 j^s}{a_D g_2^2} - \frac{2}{g_2^2}\frac{dg_2}{d\eta_2}\right) + \frac{2b_D}{a_D g_2}\frac{dg_2}{d\eta_2}\right] -$$
$$-\frac{\varepsilon}{\chi^{1/2}}\frac{k_1^2\Theta_D^2 j^p}{a_D g_2^2}\frac{dg_2}{d\eta_2} + \ldots,$$
$$z_e^p = \varepsilon\chi^{-1}k_1\Theta_D^2 j^p\,(dg_2/d\eta_2)/(a_D g_2^2) + \ldots,$$
$$I_i^p = \chi^{-1/2}k_1 j^p + \ldots,\quad I_e^p = -\chi^{-1/2}k_1 j^p + \ldots,\quad E^p = \frac{\chi^{-1/2}k_1\Theta_D j^p}{a_D g_2} + \ldots,$$
$$0 < \eta_2 = \chi^{-1/2}(\eta - \eta_D) < \infty,$$
$$g_2 = r_D\frac{(v^2 - 10v + 1)}{(v^2 + 2v + 1)},\quad v = (5 + 2\sqrt{6})\exp[(2b_D r_D/a_D)^{1/2}\eta_2].$$

Here and below the index D is assigned to values of the functions at $\eta = \eta_D$.

The expansion of the solution of the problem for the perturbations in the auxiliary transitional layer for perturbations has the form

$$z_i^p = \varepsilon^{1/4}\chi^{-3/4}C_1\exp(-Q\eta_3) + \ldots,\quad z_e^p = \varepsilon^{1/4}\chi^{-3/4}C_1\exp(-Q\eta_3) + \ldots,$$
$$I_i^p = \chi^{-1}2a_D QC_1\exp(-Q\eta_3) + \ldots,\quad I_e^p = \chi^{-1/2}k_1\,[2a_D QC_1\exp(-Q\eta_3) -$$
$$-j^p] + \ldots,\quad E^p = \frac{\Theta_D C_1\exp(-Q\eta_3)}{\varepsilon^{1/4}\chi^{3/4}\gamma}\left(\frac{4q+1}{\eta_3^2} + \frac{Q}{\eta_3}\right) + \ldots,$$
$$0 < \eta_3 = \frac{\eta - \eta_D}{(\varepsilon\chi)^{1/4}} < \infty,$$
$$Q = \left(-\frac{\alpha_1}{2a_D\Theta_D}\right)^{1/2},\quad \gamma = \left(\frac{4b_D r_D^3}{3a_D}\right)^{\frac{1}{2}},\quad q = -\frac{k_1 j^s}{4a_D\gamma},$$

where C_1 is an unknown constant, whose value is determined below; Re $Q > 0$. In the derivation of this expansion real positive values of α are excluded from the analysis.

We seek the expansions valid in the second transitional layer in the form

$$z_i^s = (\varepsilon/\chi)^{1/3} g_4(\eta_4) + \ldots, \quad z_e^s = (\varepsilon/\chi)^{1/3} f_4(\eta_4) + \ldots,$$
$$I_i^s = \frac{k_1 j^s - 2a_D\gamma}{\chi^{1/2}} + \ldots, \quad I_e^s = -\frac{k_1 j^s}{\chi^{1/2}} + \ldots, \quad E^s = \frac{E_4(\eta_4)}{\varepsilon^{1/3}\chi^{1/6}} + \ldots,$$
$$z_i^p = \varepsilon^{1/4}\chi^{-3/4} g_4^p(\eta_4) + \ldots, \quad z_e^p = \varepsilon^{1/4}\chi^{-3/4} f_4^p(\eta_4) + \ldots,$$
$$I_i^p = \chi^{-1} 2a_D Q C_1 + \ldots, \quad I_e^p = \chi^{-1/2} k_1 (2a_D Q C_1 - j^p) + \ldots,$$
$$E^p = \frac{E_4^p(\eta_4)}{\varepsilon^{5/12}\chi^{7/12}} + \ldots, \quad -\infty < \eta_4 = \frac{\eta - \eta_D}{\varepsilon^{1/3}\varepsilon^{1/6}} < \infty.$$

The boundary conditions in the limit $\eta_4 \to -\infty$ are that the functions g_4 and f_4 decay and the solution of the problem for perturbations does not grow exponentially. Analysis shows that the asymptotic expansions of the solutions of the problems in the limit $\eta_4 \to -\infty$ are:

$$g_4 = (2q+1)\gamma^{1/2}\Theta_D(-\eta_4)^{-1/2} + \ldots, \quad f_4 = 2q\gamma^{1/2}\Theta_D(-\eta_4)^{-1/2} + \ldots,$$
$$E_4 = -2(-\gamma\eta_4)^{1/2} + \ldots, \quad g_4^p = \gamma^{-1/2}(-\eta_4)^{-3/2}\Theta_D C_1(2q+1)/2 + \ldots,$$
$$f_4^p = \gamma^{-1/2}(-\eta_4)^{-3/2} q\Theta_D C_1 + \ldots, \quad E_4^p = C_1(-\gamma\eta_4)^{-1/2} + \ldots$$

The expansions valid in the third transitional layer are

$$z_i^s = \varepsilon^{1/2}\chi^{-1/2}\frac{\Theta_D G_5}{a_D E_5} + \ldots, \quad z_e^s = \varepsilon^{1/2}\chi^{-1/2}\frac{k_1 j^s \Theta_D}{a_D E_5} + \ldots,$$
$$I_i^s = \chi^{-1/2} G_5(\eta_2) + \ldots, \quad I_e^s = -\chi^{-1/2} k_1 j^s + \ldots, \quad E^s = \varepsilon^{-1/2} E_5(\eta_2) + \ldots,$$
$$z_i^p = \varepsilon^{1/2}\chi^{-1} g_5^p(\eta_2) + \ldots, \quad z_e^p = \varepsilon^{1/2}\chi^{-1} f_5^p(\eta_2) + \ldots,$$
$$I_i^p = \chi^{-1} G_5^p(\eta_2) + \ldots, \quad I_e^p = \chi^{-1/2} F_5^p(\eta_2) + \ldots, \quad E^p = (\varepsilon\chi)^{-1/2} E_5^p(\eta_2) + \ldots,$$
$$-\infty < \eta_2 < 0, \quad G_5 = 2b_D r_D^2 \eta_2 + k_1 j^s - 2a_D\gamma,$$
$$E_5 = -\left(2b_D r_D^2 \eta_2^2 / a_D - 4\gamma\eta_2\right)^{1/2}.$$

The asymptotic behavior of the solution for perturbations in the limit $\eta_2 \to -\infty$ is determined by the sign of the quantity Re $\alpha_1/p - 1$. Since for purposes of this work we are mainly interested in the harmonics (Re $\alpha = 0$) and the growing (Re $\alpha < 0$) perturbations, we shall assume below that this quantity is negative. Then the asymptotic expansions for perturbations in the limit $\eta_2 \to -\infty$ will be

$$g_5^p = O(\eta_2^{-2}), \quad f_5^p = -k_1 j^s a_D^2 \Theta_D C_1 C_2 / (p^3 \eta_2^2) + \ldots,$$
$$G_5^p \to a_D C_1 C_2, \quad F_5^p \to k_1(1 - \alpha_1/p) a_D C_1 C_2 - k_1 j^p,$$
$$E_5^p \to a_D p^{-1} C_1 C_2, \quad p = (2a_D b_D)^{1/2} r_D,$$

where C_2 is a known quantity (for brevity, the method by which it is determined is not presented).

We seek the expansions valid in the Debye layer ($0 < \eta < \eta_D$) in the form

$$z_i^s = \varepsilon^{1/2}\chi^{-1/2} g_6(\eta) + \ldots, \quad z_e^s = \varepsilon^{1/2} f_6(\eta) + \ldots,$$
$$I_i^s = \chi^{-1} G_6(\eta) + \ldots, \quad I_e^s = \chi^{-1/2} F_6(\eta) + \ldots, \quad E^s = (\varepsilon\chi)^{-1/2} E_6(\eta) + \ldots,$$
$$z_i^p = \varepsilon^{1/2}\chi^{-1/2} g_6^p(\eta) + \ldots, \quad z_e^p = \varepsilon^{1/2} f_6^p(\eta) + \ldots,$$
$$I_i^p = \chi^{-1} G_6^p(\eta) + \ldots, \quad I_e^p = \chi^{-1/2} F_6^p(\eta) + \ldots, \quad E^p = (\varepsilon\chi)^{-1/2} E_6^p(\eta) + \ldots$$

The stationary problem in the Debye layer is:

$$G_6 = a g_6 E_6/\Theta, \quad F_6 = -\mu_6 f_6 E_6,$$
$$G_6' = 2b(r^2 + c_6 f_6), \quad F_6 = k_1(G_6 - j^s), \quad \Theta E_6' = g_6; \tag{3.1}$$
$$\eta \to \eta_D: \quad g_6 \to \frac{p\Theta_D}{a_D}, \quad f_6 = \frac{k_1 j^s \Theta_D}{p(\eta - \eta_D)} + \ldots, \tag{3.2}$$

$$G_6 = 2b_D r_D^2 (\eta - \eta_D) + \ldots, \quad F_6 \to -k_1 j^s, \quad E_6 = \frac{p}{a_D}(\eta - \eta_D) + \ldots;$$

$$\eta = 0: \quad f_6 = 0, \quad c_6 = w_1(E_6^2), \quad \mu_6 = (a/\Theta)\, w_2(E_6^2). \tag{3.3}$$

It can be shown that the boundary conditions (3.2), taking into account Eqs. (3.1), are equivalent to the following two relations:

$$G_6(\eta_D) = E_6(\eta_D) = 0. \tag{3.4}$$

Thus the stationary problems for the Debye layer (3.1), (3.3), and (3.4) is closed: we have three boundary conditions to determine two constants of integration and the constant η_D. This problem in general does not have an analytic solution, and must be solved numerically; in this work the iteration algorithm of [1] is employed.

The problem for perturbations in the Debye layer is:

$$G_6^p = (a/\Theta)\left(g_6^p E_6 + g_6 E_6^p\right), \tag{3.5}$$

$$F_6^p = -\mu_6\left(f_6^p E_6 + f_6 E_6^p\right) - 2a\Theta^{-1} w_4 E_6^2 f_6 E_6^p,$$

$$-(\alpha_1/\Theta)\, g_6^p + \left(G_6^p\right)' = 2b\left(c_6 f_6^p + 2w_3 E_6 f_6 E_6^p\right),$$

$$F_6^p = k_1\left(G_6^p - j^p - \alpha_1 E_6^p\right), \quad \Theta\left(E_6^p\right)' = g_6^p;$$

$$\eta \to \eta_D: \quad g_6^p = o\left[(\eta - \eta_D)^{-1}\right], \quad f_6^p = -\frac{k_1 j^s a_D^2 \Theta_D C_1 C_2}{p^3 (\eta - \eta_D)^2} + \ldots, \tag{3.6}$$

$$G_6^p \to a_D C_1 C_2, \quad F_6^p \to k_1\left(1 - \frac{\alpha_1}{p}\right) a_D C_1 C_2 - k_1 j^p, \quad E_6^p \to a_D C_1 C_2 / p;$$

$$\eta = 0: \quad f_6^p = 0 \tag{3.7}$$

(w_3 and w_4 are derivatives of the functions $w_1(x)$, $w_2(x)$).

The joining conditions (3.6) are equivalent to the condition

$$\eta \to \eta_D: \quad E_6^p = O(1). \tag{3.8}$$

The problem (3.5), (3.7), and (3.8) is closed; it can be solved (in the general case numerically) and the heretofore unknown constant C_1 can then be determined. For brevity, we do not write out the expansions valid in the diffusion layer (with $\eta = O[(\varepsilon\chi)^{1/2}]$).

We shall examine some characteristic features of the solutions of the stationary problem. Evaluating the voltage drop in each of the asymptotic regions we find that the Debye layer makes the determining contribution to the total drop in the boundary layer. For this reason, to calculate the current-voltage characteristic of the boundary layer in the first approximation it is sufficient to solve the problem (3.1), (3.3), and (3.4).

It is interesting to compare our results with the results of [1, 7], obtained in the limit $\varepsilon \to 0$, $\chi \to 0$, and for fixed β. It can be shown that the problem (3.1), (3.3), and (3.4) can be obtained by passage to the limit $\beta \to 0$ from the corresponding problem [7] (if the latter is supplemented, as done in [1], with terms that take into account the direct ionization and the dependence of the electron mobility on the electric field intensity), so that the current–voltage characteristics will be identical with accuracy up to 15 β. On the other hand, the asymptotic structure of the transitional zone between the equilibrium region and the Debye layer is generally speaking very different.

If the direct ionization is neglected, then the ion density distribution, the ion diffusion flux density, and the electric field intensity in the Debye layer can be found neglecting the presence of electrons, after which the electron density distribution and the electron diffusion flux density can be found. This result agrees with the proposition of [8], in which the presence of electrons was ignored in the formulation of the system of equations describing the Debye layer with ionization by an external source. The problems (3.1), (3.3), and (3.4) can then be solved in quadratures.

The interesting features of the solutions obtained for the perturbations are the nonmonotonic character of the change in the orders of magnitude of the functions sought as well as the nonquasineutrality of the solutions in all regions, except the auxiliary transitional layer (in particular, the perturbations are nonquasineutral even in the equilibrium region).

The Debye layer makes the determining contribution to the perturbations of the total voltage drop. The system (3.5), describing the distribution of perturbations in the Debye layer, is analogous to the system of equations for the stationary solution (3.1) (in the sense that it can be obtained by introducing nonstationary terms into (3.1) and linearizing). The boundary conditions at the walls are also analogous. The difference in the form of the boundary conditions in the limit $\eta \to \eta_D$ is linked with the fact that the thickness of the Debye layer is a function of the current density.

Defining the impedance of the boundary layer R^0 as the ratio of the perturbations of the voltage drop to the perturbation of the current density, in the first approximation we find

$$R^0 = [\delta k T_\infty/(e^2 D_{i\infty} n_{er\infty})]R; \tag{3.9}$$

$$R = \left(\frac{\chi}{\varepsilon}\right)^{1/2} \frac{1}{j^p} \int\limits_0^{\eta_D} E_6^p d\eta, \tag{3.10}$$

where E_6^p is the solution of the problem (3.5), (3.7), and (3.8), which, generally speaking, must be found numerically.

If direct ionization is eliminated from the analysis, then the equations for the perturbations in the Debye layer, like the stationary equations, separate, and their solution can be constructed in quadratures. If further simplifications are made, namely, it is assumed that $br^2 \equiv$ const and $a \equiv$ const for $\eta \le \eta_D$, then these integrals can be calculated analytically. As a result $R = (\chi/\varepsilon)^{1/2}\eta_D/(p - \alpha_1)$.

We shall write this formula in the dimensionless variables in the form

$$R^0 = \frac{R_d^0 R_C^0}{R_d^0 + R_C^0}, \quad R_d^0 = \frac{dU^s}{dj^{s0}}, \quad R_C^0 = -\frac{4\pi y_D}{\lambda} \tag{3.11}$$

(U^s, j^{s0}, and y_D are the dimensional values of the stationary voltage drops in the Debye layer, the current density, and the thickness of the layer, respectively). It follows from here that in this particular case the impedance of the layer in the first approximation equals the complex resistance of the circuit, consisting of resistances connected in parallel and equal to the differential resistance of the layer R_d^0 and a capacitor with a gap y_D between the plates.

We shall present also the asymptotic form of the formulas (3.9) and (3.10) for large values of α_1. This form can be obtained from the asymptotic solution of the problem (3.5), (3.7), and (3.8) (in the first approximation the function E_6^p then turns out to be constant, equal to $-j^p/\alpha_1$):

$$R = -(\chi/\varepsilon)^{1/2}\eta_D/\alpha_1, \quad R^0 = R_C^0. \tag{3.12}$$

The case of perturbations with the characteristic time of the order of the electron drift time $\alpha = O(\varepsilon^{-1/2}\chi^{-1})$ can be studied analogously to the foregoing case. Without indicating the detailed results, we note that a formula, identical to (3.12), is obtained for the impedance.

We shall make a remark of a methodical character. In many cases the first terms of the adjoining asymptotic expansions of some functions do not join together. It is important, however, that each of the indicated first terms, being transformed to the adjoining variable, is of an order less than that of the first term of the adjoining expansion. The suggests that the first terms of the indicated expansions join with the terms of the adjoining expansions whose order is higher than that of the first expansion. Based on this proposition it may be expected that the given results are adequate for practical applications; in particular, the asymptotic representations of the functions in the intermediate regions are found by adding the first asymptotic terms of the adjoining expansions. From the methodical standpoint, in order to join the expansions and check this proposition, either higher order approximations or the intermediate limit must be employed.

4. Results for the Impedance. The formula (3.10) can be regarded as uniformly applicable with all indicated values of α. In [2-6], within the framework of the model taking into

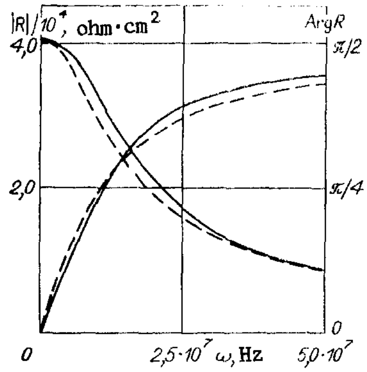

Fig. 1

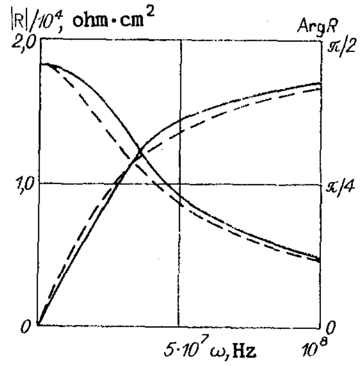

Fig. 2

account the presence of only ions in the Debye layer, it is concluded that in the range of frequencies exceeding significantly the inverse drift time of ions drifting through the Debye layer, the impedance of the cathode layer becomes equal to R_C^0. The relations (3.12) show that this result remains valid also in the present model, in which the presence of electrons in the Debye layer is also significant.

The impedance calculated based on the formulas (3.9) and (3.10) for the following conditions [1] is shown by the solid lines in Figs. 1 and 2: the plasma consists of the combustion products with a potassium additive and atmospheric pressure; the molar fraction of potassium atoms across the Debye layer is constant and equals 1%; the profile shown in Fig. 2 of [1] is taken for the distribution of the plasma temperature; $\omega = i\lambda$; the calculations were performed for $j^{s0} = 5.3$; 16.6 mA/cm² and U^s = 311 and 606 V (Figs. 1 and 2, respectively).

It is interesting to note that the results of the numerical calculation can be adequately approximated by the formulas (3.11) (broken lines).

It follows from the foregoing that measurement of the impedance at frequencies of the order of the inverse drift time of ions through the Debye layer enables determining the thickness of the Debye layer corresponding to the given current density. Thus comparison of the theory of current flow through the cathodic boundary layer with experiment can be performed not only based on the usually measured current-voltage characteristic $U^s(j^{s0})$, but also based on the dependence $y_D(j^{s0})$, which would undoubtedly be very useful. For example, even in the simplest case, when the effect of direct ionization is insignificant, it is often difficult to make a direct comparison of the theoretical dependence $U^s(j^{s0})$ with experiment owing to the uncertainty in the distribution of the rate of stepped ionization in the cathodic region. The possibility of comparing at the same time $y_D(j^{s0})$ greatly simplifies this situation. For example, the distribution of the rate of stepped ionization can be found from an analysis of this dependence, determined in the experiment: $f_{i1} = -[e(dy_D/dj^{s0})]^{-1}$; then using this dependence the dependence $U^s(j^{s0})$ can be compared.

It is known from experiments with cold cathodes that at some value of the current density the diffusion form of the discharge transforms into an arc form; this transition is probably the result of the development of some instability. Based on the results of the analysis of experimental data, performed in [9], it is natural to examine, as a first step in the development of the corresponding theory, the possibility that this instability exists within the framework of the present model, i.e., neglecting emission from the surface and the effect of Joule heating on the temperature of the plasma. Since the one-dimensional perturbations do not change the voltage drop in the boundary layer, the characteristic frequencies (more precisely, the characteristic decrements) for such perturbations correspond to zeroes of the impedance, i.e., they are roots of the equation $R(\alpha) = 0$. If it is assumed that the one-dimensional perturbations are the most dangerous perturbations (they develop with the lowest values of the current density), then the appearance of instability corresponds to the appearance of a root of this equation in the left half-plane. The calculation of the function $R(\alpha)$ for the above-indicated conditions showed, however, that there are no zeros in this half-plane, i.e., within the framework of the model studied the instability apparently does not occur.

LITERATURE CITED

1. M. S. Benilov, G. A. Lyubimov, et al., "Effect of a superstrong field on the characteristics of the cathode layer in a molecular-gas plasma," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1984).
2. B. M. Oliver, P. R. Smy, and R. M. Clements, "Small-signal transit time analysis of a planar mobility-dominated ion sheath," J. Appl. Phys., 45, No. 12 (1974).
3. B. M. Oliver, R. M. Slements, and P. R. Smy, "The effect of relaxation of the quasineutral region on the impedance of a planar probe in a static collisional plasma," J. Phys. D, 9, No. 12 (1976).
4. E. F. Prozorov and K. N. Ul'yanov, "Impedance of a flat probe in nonisothermal locally collisional plasma," Teplofiz. Vys. Temp., 21, No. 3 (1983).
5. E. F. Prozorov and K. N. Ul'yanov, "Impedance of a cylindrical probe in a nonisothermal plasma," Teplofiz. Vys. Temp., 21, No. 6 (1983).
6. E. F. Prozorov, "Probe diagnostics of a dense plasma of self-sustained and non-self-sustained discharges with probe potential modulation," Author's Abstract of Candidate's Disseration in Physicomathematical Sciences, High-Energy Institute, Moscow (1984).
7. M. S. Benilov, "Near-electrode zone in a weakly ionized plasma in chemical equilibrium" Izv. Akad. Nauk SSSR, MZG, No. 1 (1982).
8. E. F. Prozorov, Ya. I. Londer, et al., "Probe method for diagnostics of a non-self-sustained discharge plasma," Teplofiz. Vys. Temp., 18, No. 1 (1980).
9. G. A. Lyubimov, "Conditions for breakdown of the near-cathode layer in ionized gas flow," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1973).